NEMO 2008 Problems - Individual Round

1. Albert buys 100 shares of NEMO stock at \$45.00 per share. Nitin buys 200 shares at \$51.00 per share. What was the average cost per share of the two purchases?

2. Two standard dice are rolled. What is the probability that the dice show a total of seven pips (dots)?

3. On Sunday, Jacob catches bird flu. He unknowingly goes to school Monday and infects five people. Those five people go to school Tuesday and each infect five uninfected people. There are 2600 people at Jacob's school. By what day is the whole school infected, assuming all students come to school on weekends as well? Note: People who get infected only go to school for one day while infected.

4. Compute $\frac{17!\cdot 13!\cdot 7!}{16!\cdot 14!\cdot 5!}$. Note: $n! = n\cdot(n-1)\cdot(n-2)\cdots 3\cdot 2\cdot 1$

5. The NEMO committee is being formed by Vivek, the NEMO president. There are six boys and five girls who want to be on the committee. There are four spots on the committee. At least one committee member must be a boy, and at least one must be a girl. How many possible ways can Vivek choose the committee?

6. Mary eats half of a pizza. Ned eats one-third of the pizza. Oscar eats one-seventh of the pizza. Paul eats the rest of the pizza. What is the ratio of how much pizza Mary eats to how much pizza Paul eats?

7. Compute $0-1+2-3+4-5+\cdots-99+100$.

8. An equilateral triangle has side length s. A line segment is drawn from one vertex to the midpoint of the opposite side. What is the ratio between the length of that line segment and the length of a side of the triangle?

9. A right triangle has legs of length 12 and 35. Find the area of the circle circumscribed about the triangle.

10. A cube has side length $\sqrt{6}$. Find the sum of the volume, surface area, length of one face diagonal, and length of one space diagonal.

11. A right cylinder has a lateral surface area of 2π. The area of one end of the cylinder minus twice the circumference of that end of the circle is equal to -4π. Find the volume of the cylinder.

12. A square pyramid has a base with area 16. The segment from the top of the pyramid to a corner of the square has length 8. Find the surface area of the pyramid.

13. A geometric series has first term 16 and common ratio $\frac{1}{2}$. What is the integer nearest to the sum of the first ten terms of the geometric series?

14. How many real zeroes does $f(x) = x^3 + x^2 + 2x + 2$ have?

15. Three circles are all tangent to each other. One has radius 3, one has radius 5, and one has radius 12. What is the area of the triangle with vertices at the centers of the three circles?

16. Let $\phi(n)$ be the number of positive integers a less than or equal to n such that the greatest common divisor of a and n is 1. Find $\phi(20)$.

17. Express the following as a common fraction:

$$1+\frac{2}{3+\frac{4}{5+\frac{6}{7}}}$$

18. Unfortunately for his budget, Alex often incurs speeding tickets. His last ticket cost \$16.50. How many ways can Alex pay his speeding ticket in quarters and dimes?

19. Let $\sigma(n)$ be the sum of all of the positive integer divisors of n, including n. Compute $\sigma(100)$.

20. $3^a = 64$ and $4^b = 81$. Compute ab.

21. Find $\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}$.

22. Let $a \diamond b$ denote $ab + a + b$. Compute $(2 \diamond 3) \diamond (4 \diamond 5)$.

23. Call a positive integer *vivacious* if the sum of its digits in base eight is divisible by seven. How many vivacious integers are greater than 1 (base 8) and less than 76 (base 8)?

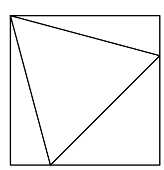
24. A triangle has sides of length 2, 3, and 4. The radius of a circle inscribed in the triangle is $\frac{1}{6}\sqrt{15}$. Find the area of the triangle.

25. Let $\nabla n = 2^{2^n} + 1$. Find the number of prime factors of $\nabla(\nabla 0)$.

26. What is the least integer a such that $\sqrt[4]{a} > 1 + \sqrt{2}$?

27. Hyunji has eight cookies. She walks through the Cookie Monster Machine (CMM) four times. Each time she walks through, there is a $\frac{1}{4}$ chance that she will lose one cookie, a $\frac{1}{2}$ chance that she will neither gain nor lose a cookie, and a $\frac{1}{4}$ chance that she will gain one cookie. What is the probability that Hyunji has six cookies after her encounters with the CMM?

28. An equilateral triangle is drawn inside a square as shown. What is the ratio of the area of the triangle to the area of the square?



29. Let a and b be the solutions to the quadratic equation $11x^2 - 37x + 23 = 0$. What is the value of $(a+1)(b+1)$?

30. The Fibonacci series is defined by $F_0 = F_1 = 1$ and $F_{n+2} = F_{n+1} + F_n$. How many of $\{F_2, F_3, F_4, \cdots, F_{20}\}$ are divisible by 5?