

NEMO 2008 - Team Round

1. Abhiram has a die with the following faces: 1,1,2,3,5,8. He rolls the die twice. What is the probability that the sum of the two rolls is greater than 11?
2. A triangle has vertices $(10,1)$, $(2,11)$, and $(3,4)$. Compute the area of the triangle.
3. In how many ways can I pay a fee of 47 cents using standard U.S. currency?
4. Find all integers for which $x^4 - 4x^3 + 5x^2 + 2x + 1 \leq 0$.
5. The ratio of the volume of a sphere to its surface area is 2. Find the volume of the sphere.
6. Alice, Bob, Cathy, Don, Eve, and Fred go out for a movie. They sit in arbitrary order in six adjacent seats. What is the probability that Alice is sitting next to Fred?
7. How many ordered triplets (a, b, c) of positive integers satisfy $a + b + c = 8$?
8. Let $ABCDE$ be a regular pentagon with side length 1 and center O . Given that $AO = s$, compute AC in terms of s .
9. Let C be a cube. Let T be the polyhedron formed by joining the six centers of the faces of C . Compute the ratio of the volume of T to the volume of C .
10. The Fibonacci sequence is given by $F_0 = F_1 = 1$ and $F_{n+2} = F_{n+1} + F_n$. What is the smallest positive integer n that cannot be written as the sum of at most three Fibonacci numbers?