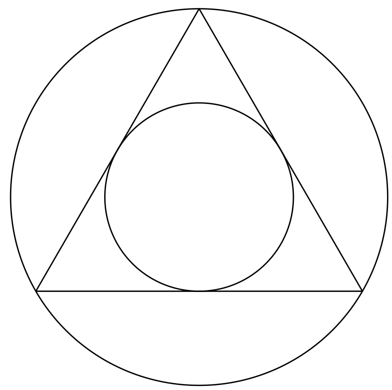
NEMO 2008 - Tiebreaker Round

1. What is the greatest integer k such that 3^k divides $100!$?

2. How many triangles exist with integer side lengths and perimeters at most seven?

3. Find the ratio of the radius of the smaller circle to the radius of the larger circle, given that the triangle is equilateral.



4. Compute $1453 \cdot 1459 \cdot 1471 - 1451 \cdot 1463 \cdot 1469$.

5. The Catalan recurrence is given by the initial conditions $C_0 = C_1 = 1$ and the recurrence $C_{n+1} = C_0C_n + C_1C_{n-1} + C_2C_{n-2} + \cdots + C_{n-1}C_1 + C_nC_0$. Compute $C_0 + C_1 + \cdots + C_7$.

6. Let r_1, r_2, r_3, r_4, r_5 be the roots of the quintic polynomial $x^5 + 2x^4 - 5x^3 - 3x^2 + 4x + 1$. Compute $r_1^3 + r_2^3 + r_3^3 + r_4^3 + r_5^3$.

7. Point P is inside square $ABCD$ such that $PA = PB = PM$, where M is the midpoint of segment CD. Compute the ratio of the area of triangle PAB to the area of quadrilateral $PADM$.